Table 2 Results at α = 7.84 deg

	C_L			C_M			_C_M/C_L	
Results	1:40	1:32	Difference	1:40	1:32	Difference	1:40	1:32
Experimental	0.5356	0.5599	0.0243	_0.08057	_0.08492	0.00435	0.1504	0.1517
Correction term	0.0252	0.0418		_0.01870	_0.02200			
Corrected result	0.5104	0.5181	0.0077	_0.06187	_0.06292	0.00105	0.1212	0.1214

Table 3 Results at α = 3.74 deg

	C_L			C_M			_C_M/C_L	
Results	1:40	1:32	Difference	1:40	1:32	Difference	1:40	1:32
Experimental	0.2431	0.2555	0.0124	_0.04714	_0.04817	_0.00103	0.1939	0.1885
Correction term	0.0078	0.0145		_0.00127	_0.00260			
Corrected result	0.2353	0.2410	0.0057	_0.04587	_0.04557	0.00030	0.1949	0.1891

after the correction, as earlier observed, is also related to experimental errors in force measurements and model position.

For the pitching moment, the accuracy of the corrected values appears satisfactory, in that the corrected estimation of the lift point of application is practically the same for the two models.

The results for a lower angle of attack (3.74 deg) are shown in Table 3. This condition is clearly characterized by a lower wall interference effect, and this leads to a greater sensitivity to the measurement uncertainty (both for the forces and the wall pressure). Indeed, when the results are compared with those of the preceding analyzed condition, it is evident that the lift coefficient is characterized by a lower accuracy after the correction procedure, with a difference of 2.4% between the two models. Also, the pitching moment results are less accurate: A difference of about 0.6% of the mean aerodynamic chord remains in the evaluation of the point of application of the lift.

Conclusions

A previously proposed posttest correction procedure has been applied to experimental data in subsonic low angle of attack conditions. It has been shown that the correction procedure effectively reduces the wall interference effects. However, as expected, the correction becomes more accurate when the wall effects to be corrected are important. Therefore, great care must be taken in deciding when to apply the proposed correction procedure: Indeed, for low blockage factors and low angles of attack, when the wall effects are very small, it is possible that measurement errors in the wall pressure evaluation produce errors in the correction procedure greater than the correction term itself.

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Analytical Prediction of Panel Flutter Using Unsteady Potential Flow

N. Soltani* and V. Esfahanian†
University of Tehran, 11365-4563 Tehran, Iran
and
H. Haddadpour‡
Sharif University of Technology, 11365-9567 Tehran, Iran

Introduction

DURING the last 50 years, many investigators have made significant contributions to the study of panel flutter. These authors have considered many aspects of the flutter models and a wide variety of aerodynamic theories. Many structural and aerodynamic issues such as the plate modeling, initial stresses, thermal effects, large deflection,[1–3] piston theory, unsteady potential flow and viscous flow effects,[4] respectively, have been considered. For instability prediction the analytical[1] and computational[5–7] methods have been also developed.

Because of the complexity of the problem, few analytical solutions are available in the literature. All of the analytical solutions use the piston theory for the aerodynamic modeling. This theory have been developed by the application of power series expansion in unsteady potential flow and retention of only the first two terms.[4]

In the present study full unsteady potential flow aerodynamics are applied to predict panel flutter analytically, using the modal analysis technique.

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*Associate Professor, Mechanical Engineering Department; nsoltani@ut.ac.ir.

†Associate Professor, Mechanical Engineering Department; evahid@ut.ac.ir.

‡Assistant Professor, Department of Aerospace Engineering; haddadpour@sharif.edu.

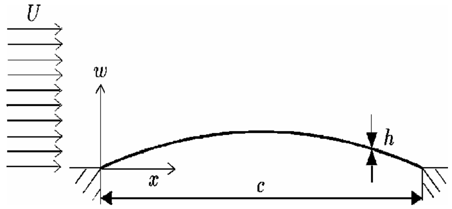

Fig. 1 Two-dimensional plate in supersonic flow.

Formulation

An isotropic simply supported flat panel with length c and infinite aspect ratio in supersonic airstream with the air velocity U is shown in Fig. 1. Suppose that the plate is thin and free of initial stresses with linear behavior. Then the governing differential equation of the plate is

$$D\frac{\partial^4 w}{\partial x^4} + \rho h\frac{\partial^2 w}{\partial t^2} + p(x,t) = 0 \tag{1}$$

where $D = Eh^3/12(1-\nu^2)$ and w, p, ν, E are panel deflection, aerodynamic pressure, Poisson ratio, elastic module, and ρ and h are density and thickness of the plate, respectively.

For aerodynamic purposes, after solving the governing differential equation for the two-dimensional linear isentropic inviscid potential flow, one arrives at[8]

$$\underbrace{\phi(x,z)\exp(\alpha t)}_{\phi(x,z,t)} = \frac{-1}{\beta}\int_0^{x-\beta z}\underbrace{w_a(\xi)\exp(\alpha t)}_{w_a(\xi,t)}\exp\left[-\frac{M\alpha(x-\xi)}{a\beta^2}\right] \times J_0\left(\frac{\alpha\sqrt{(x-\xi)^2-\beta^2 z^2}}{ia\beta^2}\right)\mathrm{d}\xi \tag{2}$$

where $\beta = \sqrt{(M^2-1)}$ and J_ν, ϕ, M, a, w_a, and α are Bessel functions (first kind) of order ν, velocity potential function, Mach number, sound speed, induced velocity, and time exponent coefficient, respectively. The induced velocity and pressure distribution can be expressed as

$$w_a(x,t) = \frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} = \left.\frac{\partial \phi}{\partial z}\right|_{z=0} \tag{3}$$

$$p(x,t) = -\rho_a\left.\left(\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x}\right)\right|_{z=0} \tag{4}$$

where ρ_a is air density. Assuming the displacement is an exponential function of time that is, $w(x,t) = W(x)e^{\alpha t}$, where $\alpha = \alpha_R + i\alpha_I$, and α_R and α_I are the panel damping rate and frequency, respectively. By substituting $w_a(x,t)$ into Eq. (2) and using the result in Eq. (4) and introducing the following nondimensional quantities

$$x = x/c, \qquad W = W/c, \qquad t = Ut/c, \qquad \alpha = \alpha c/U \tag{5}$$

and

$$\lambda = 12(1-\nu^2)\frac{M^2}{\beta}\left(\frac{c}{h}\right)^3\frac{\rho_a a^2}{E}$$

$$k = \lambda\alpha\left(\frac{M^2-2}{M^2-1} + \frac{h}{c}\frac{\rho}{\rho_a}\alpha\beta\right) \tag{6}$$

Equation (1) can be rewritten as follows:

$$\frac{\mathrm{d}^4 W}{\mathrm{d}x^4} + \lambda\frac{\mathrm{d}W}{\mathrm{d}x} + \lambda\alpha\left(1 + \frac{h}{c}\frac{\rho}{\rho_a}\alpha\beta\right)W + \frac{\lambda\alpha}{\beta^2}\int_0^x \exp\left[-\frac{M^2\alpha(x-\xi)}{\beta^2}\right]\left[\alpha W(\xi) + \frac{\mathrm{d}W}{\mathrm{d}\xi}\right] \times\left\{iJ_0\left[\frac{M\alpha(x-\xi)}{i\beta^2}\right] + MJ_1\left[\frac{M\alpha(x-\xi)}{i\beta^2}\right]\right\}\mathrm{d}\xi = 0 \tag{7}$$

Using the boundary conditions of the plate, and integration by parts of Eq. (7), and defining $u = M\alpha(x-\xi)/(i\beta^2)$ yields

$$\frac{\mathrm{d}^4 W}{\mathrm{d}x^4} + \lambda\frac{\mathrm{d}W}{\mathrm{d}x} + \overbrace{\lambda\alpha\left(\frac{M^2-2}{M^2-1} + \frac{h}{c}\frac{\rho}{\rho_a}\alpha\beta\right)}^{k}W + \frac{\lambda\alpha^2}{\beta^4}\int_0^x \exp(-iMu)W(\xi)\left[\left(\frac{M^2}{2}+1\right)J_0(u) - 2iMJ_1(u) - \frac{M^2}{2}J_2(u)\right]\mathrm{d}\xi = 0 \tag{8}$$

This is a coupled integro-differential equation that describes the plate aeroelastic behavior. The first three terms of Eq. (8) are the same as the quasi-steady formulation,[1] and the integral term is the effect of unsteady flow that describes the effects of spatial and temporal memory, that is, the pressure at a particular point and at a particular time is influenced by the motion at all upstream points and at all previous times.[4] The coefficient $\lambda\alpha^2/\beta^4$ indicates that the integral term can be neglected only in low frequency and high Mach numbers.[4,8]

For determination of aeroelastic stability, Eq. (8) is solved by using the Galerkin method. In this method the solution is assumed to be a linear combination of structural natural mode shapes ψ_n, as

$$W(x) = \sum_{n=1}^{\infty} a_n\psi_n(x) \tag{9}$$

where a_n are constants and ψ_n satisfies the same boundary conditions as W and consequently an orthogonality condition holds with respect to mass and stiffness. Applying the Galerkin residual method to Eq. (8) using Eq. (9) will give

$$\int_0^1 \psi_m(x)\sum_{n=1}^{\infty} a_n\left\{\frac{\lambda\alpha^2}{\beta^4}\int_0^x \exp(-iMu)\psi_n(\xi)\left[\left(\frac{M^2}{2}+1\right)J_0(u) - 2iMJ_1(u) - \frac{M^2}{2}J_2(u)\right]\mathrm{d}\xi + \frac{\mathrm{d}^4\psi_n}{\mathrm{d}x^4} + \lambda\frac{\mathrm{d}\psi_n}{\mathrm{d}x} + k\psi_n\right\}\mathrm{d}x = 0 \tag{10}$$

This equation can be rewritten in matrix form as follows:

$$\boldsymbol{Ga} = 0 \tag{11}$$

where $\boldsymbol{a}$ is the vector of a_n constants and $\boldsymbol{G}$ is the coefficient matrix. The natural mode shapes of the corresponding plate are $\psi_n(x) = \sin(n\pi x)$; therefore, the last three terms in Eq. (10) can be integrated easily. Changing the variable of integration and defining $v = M\alpha\xi/(i\beta^2)$, the integral of the unsteady term can be expressed as the following:

$$\sum_{n=1}^{\infty} a_n\int_0^1\int_0^x \exp(-iMv)\left[\left(\frac{M^2}{2}+1\right)J_0(v) - 2iMJ_1(v) - \frac{M^2}{2}J_2(v)\right]\sin(m\pi x)\sin[n\pi(x-\xi)]\,\mathrm{d}\xi\,\mathrm{d}x \tag{12}$$

Using the modified Bessel function $I_\nu(x) = i^{-\nu}J_\nu(ix)$ and defining

$$f(\xi) = \exp(-M^2\alpha\xi/\beta^2)\left[(M^2/2+1)I_0(M\alpha\xi/\beta^2) - 2MI_1(M\alpha\xi/\beta^2) + (M^2/2)I_2(M\alpha\xi/\beta^2)\right] \tag{13}$$

leads to the coefficients of matrix $\boldsymbol{G}$ as

$$G_{mn} = \begin{cases} n\lambda \dfrac{m\left[1 - (-1)^{(n+m)}\right]}{m^2 - n^2} + \dfrac{\lambda\alpha^2}{\beta^4} \displaystyle\int_0^1 f(\xi) \dfrac{n \sin(m\pi\xi) + (-1)^{(n+m)} m \sin(n\pi\xi)}{\pi(n^2 - m^2)} \,\mathrm{d}\xi & n \neq m \\ \dfrac{(m\pi)^4 + k}{2} + \dfrac{\lambda\alpha^2}{\beta^4} \displaystyle\int_0^1 f(\xi) \dfrac{m\pi \cos(m\pi\xi) + \sin(m\pi\xi) - m\pi\xi \cos(m\pi\xi)}{2m\pi} \,\mathrm{d}\xi & n = m \end{cases} \tag{14}$$

The first term of Eq. (14) is the same as the piston theory, and the integral term is the effect of unsteady flow.

For stability study there are two different methods. The first starts from evaluating the roots of $\boldsymbol{G}$ matrix determinant α by increments of λ. This process is very complex because of the highly nonlinear behavior of the integral term and stops after the real part of α vanishes. The second method is the U-g method and merits special mention because it replaces the problem of finding the roots of a highly nonlinear equation by an easier standard matrix eigenvalue analysis. For this purpose the $\boldsymbol{G}$ matrix can be decomposed into two matrices, namely, $\boldsymbol{G} = \boldsymbol{A} - \lambda\boldsymbol{B}$. By defining $\Lambda = (1 + i\,g)/\lambda$ as a new eigenvalue, Eq. (11) can be rewritten as

$$\Lambda \boldsymbol{A}\boldsymbol{a} = \mathbf{Ba} \tag{15}$$

The U-g method of flutter analysis (or $\lambda - g$ method) is a looping procedure. The values of λ and g are solved for various imaginary values of α. The solutions are not valid except when $g = 0$ and sinusoidal motion of the system exists. Plots of g vs λ can be used to determine the flutter boundaries, where g goes through zero to positive values.

Numerical Results

The application of this study is considered to compute the eigenvalues of a typical two-dimensional plate in a supersonic airstream flow.

The value of g vs λ for the present method is compared with the piston theory results in Fig. 2. The corresponding value of λ where g vanishes is the critical value. Figure 2 shows the results of the piston theory are close to those of the unsteady potential flow method.

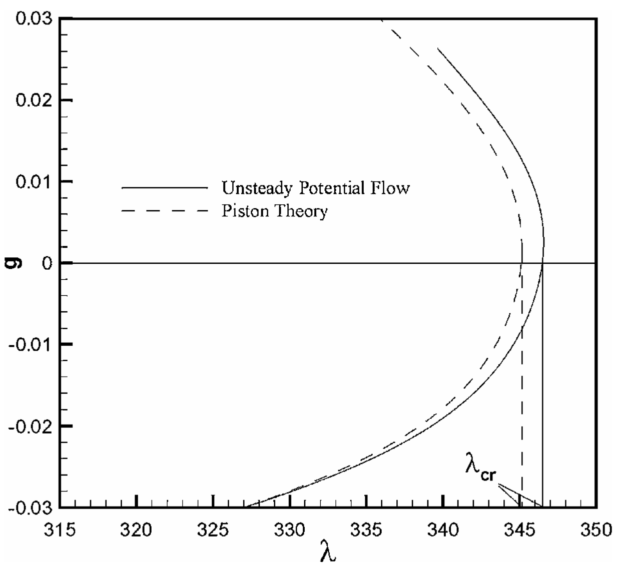

Fig. 2 *g* **vs λ.**

The result of the present work, along with the results of other references, are tabulated here, using six natural mode shapes for modal analysis: λ_{cr} modal by unsteady potential flow, 347; modal by piston theory [elimination of integral terms in Eq. (14)], 345; finite element method by piston theory,[2] 342; and exact by piston theory,[1] 343. Note that the result of unsteady aerodynamic simulation is slightly different from the results of quasi-steady aerodynamic models. But the present results shows the validity of quasi-steady models for panel flutter prediction as a conservative engineering estimate.

Conclusions

An analytical procedure for supersonic flutter analysis of two-dimensional panels has been developed using unsteady potential flow aerodynamic theory. Unlike the research of others, herein the authors considers a full unsteady potential flow aerodynamic model instead of quasi-steady aerodynamics for analytical prediction of panel flutter. A solution procedure for the governing fourth-order integro-differential equation is presented in terms of the natural mode shapes of the plate. The local spatial influence is considered as an integral over the plate area instead of using an integral approach that uses the Mach cone approach at a specified point. Because of the high nonlinearity of the problem with respect to the time exponent coefficient, the U-g method is used for flutter prediction. Results indicate a stabilizing effect of the unsteady potential flow aerodynamics theory compared to the quasi-steady first-order piston theory, which is usually employed as a conservative estimate.

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